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Phase-Plane Trajectories of EEG Seizure Patterns in Epilepsy

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ABSTRACT. *We investigated EEG phase-plane trajectories associated with the transition from the interictal state to clinical seizures. Our recordings were taken from scalp and invasive electrodes placed in epilepsy patients undergoing prolonged video/EEG monitoring. We systematically examined trajectories from 60 seizures in an attempt to determine criteria which may be used for classifying interictal and ictal patterns. These phase-plane trajectories emphasize the presence of repetitive waves, and clearly show some phase relationships between EEG channels. It is possible to follow the progress of seizure onset using these trajectories, and to define the time-course of the activity. We have found that phase-plane representations provide a different set of criteria for classifying and grouping EEG signals.*

KEYWORDS. *Chaos, electroencephalography, epilepsy, nonlinear dynamics.*

INTRODUCTION

The phase-plane provides a way of displaying time-series data in a form that emphasizes the dynamic activity in a system. This display is achieved by defining the axes so that the usual "wave" representation is transformed into a

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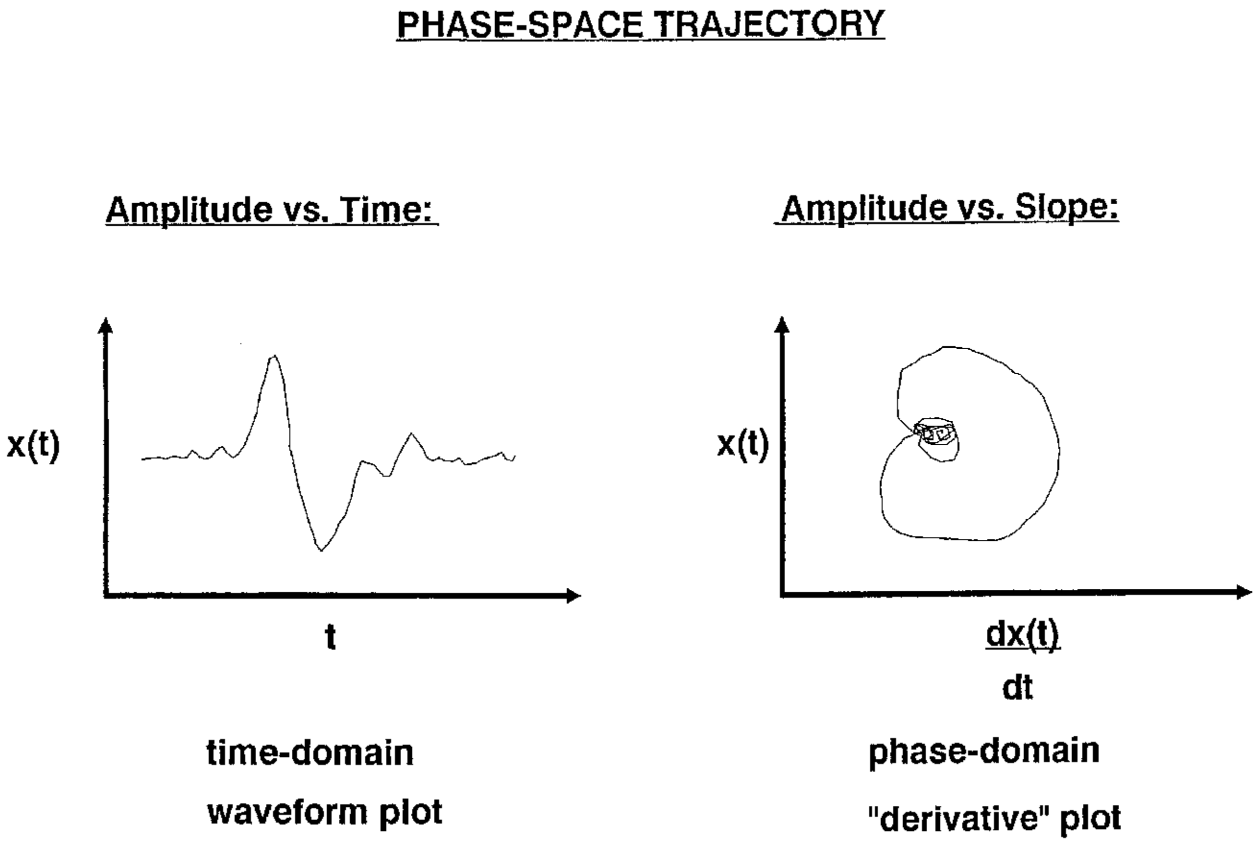

FIG. 1. Conventional time-series plot and phase-space trajectory.

trajectory that looks like a two or three-dimensional object. These plots are similar to the "Lissajous" figures familiar to engineers who study electronic signals using the oscilloscope.

The phase-plane trajectory is calculated from the raw waveform data. In the method we have chosen, the vertical axis is amplitude exactly as in conventional time-series displays. However, the horizontal axis is redefined so that rather than using time, we use the first derivative of the signal, which is the rate of change of the signal per unit time (slope). This axis is computed by calculating the difference between successive data points, and using this value as the horizontal axis instead of the actual value of the time coordinates.

Figure 1 illustrates how this representation converts a typical wavelet from amplitude-vs-time form into a trajectory in the phase-domain plot. Mathematically, a phase-plane trajectory can be thought of as capturing the dynamical state of a system, by defining a path through the phase space that defines the system. This definition is valid only for simple, time-invariant systems such as oscillators or pendulums. It has the strength of being equally useful for nonlinear as well as linear systems, and the brain/EEG generator is understood to be a nonlinear system (Roschke and Basar 1988). A phase-plane trajectory can be thought of as valid for only the time period while the system is stable. However, the EEG is produced by a highly time-varying system, so that a given phase-

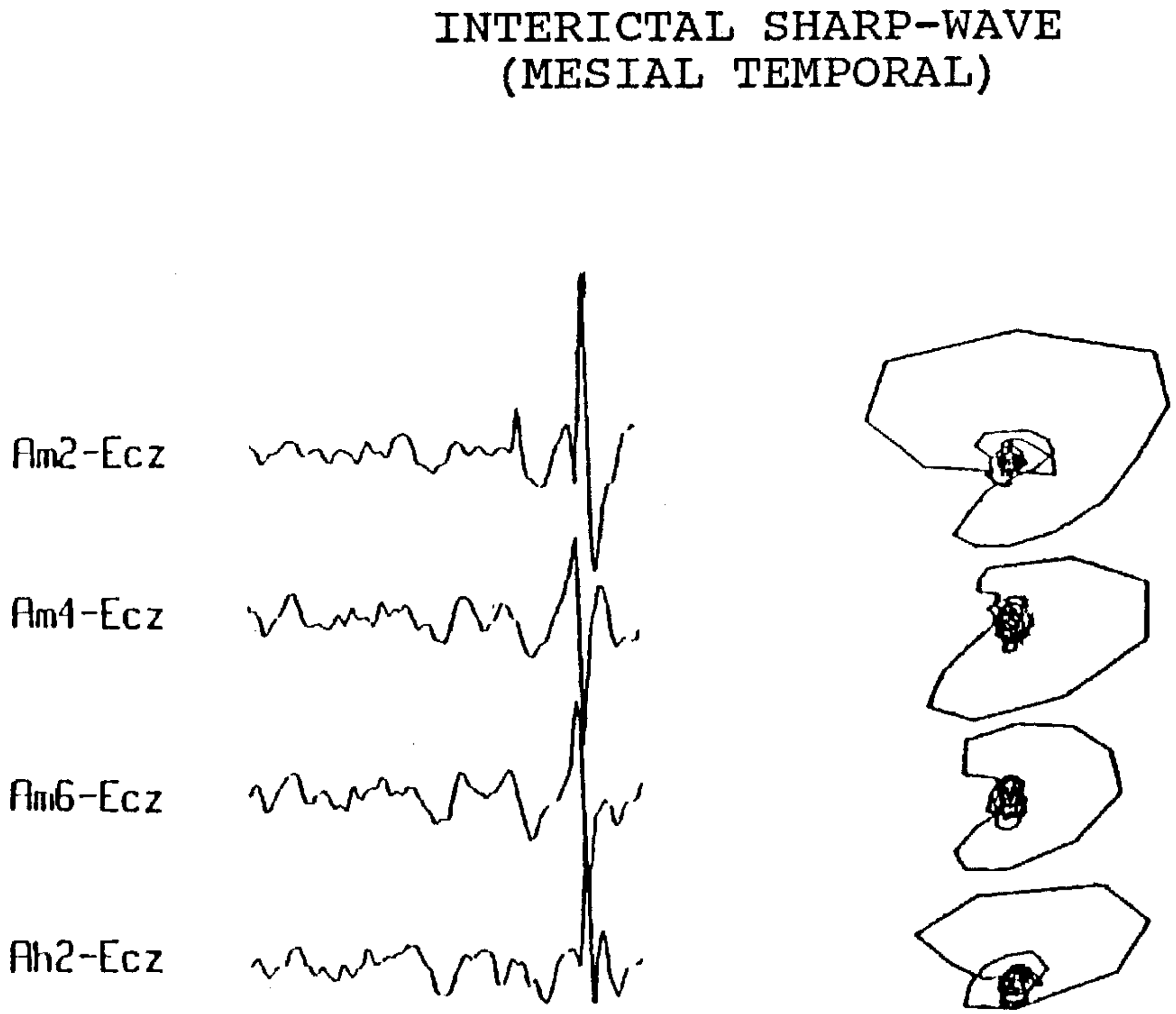

FIG. 2. EEG and phase-space trajectory of a sharp wave from limbic depth electrodes.

plane plot will represent the system for only the short time during which the brain/EEG system dynamics do not change.

Several examples will serve to illustrate the phase-plane trajectories of some typical EEG signals. Figure 2 shows an interictal sharp wave, recorded from a depth electrode, in both time and phase domains. The large trajectories due to the sharp, high-amplitude component are clearly evident. The phase inversion between channels is also evident in the orientation of the shapes. Figures 3 and 4 show the appearance of muscle and eyeblink artifact, respectively. Each activity exhibits a unique appearance in the phase domain.

We also examined the appearance of trajectories associated with the onset of epileptic seizures. Figure 5 shows eight successive 10-second epochs containing the onset of a seizure recorded with depth electrodes. As the seizure sets in, conspicuous shapes emerge, whose shape and orientation are related to the details of the epileptiform fast activity. As an illustration of the short-term variations in such plots, Figure 6 shows a similar onset, shown as 20 successive 1-second

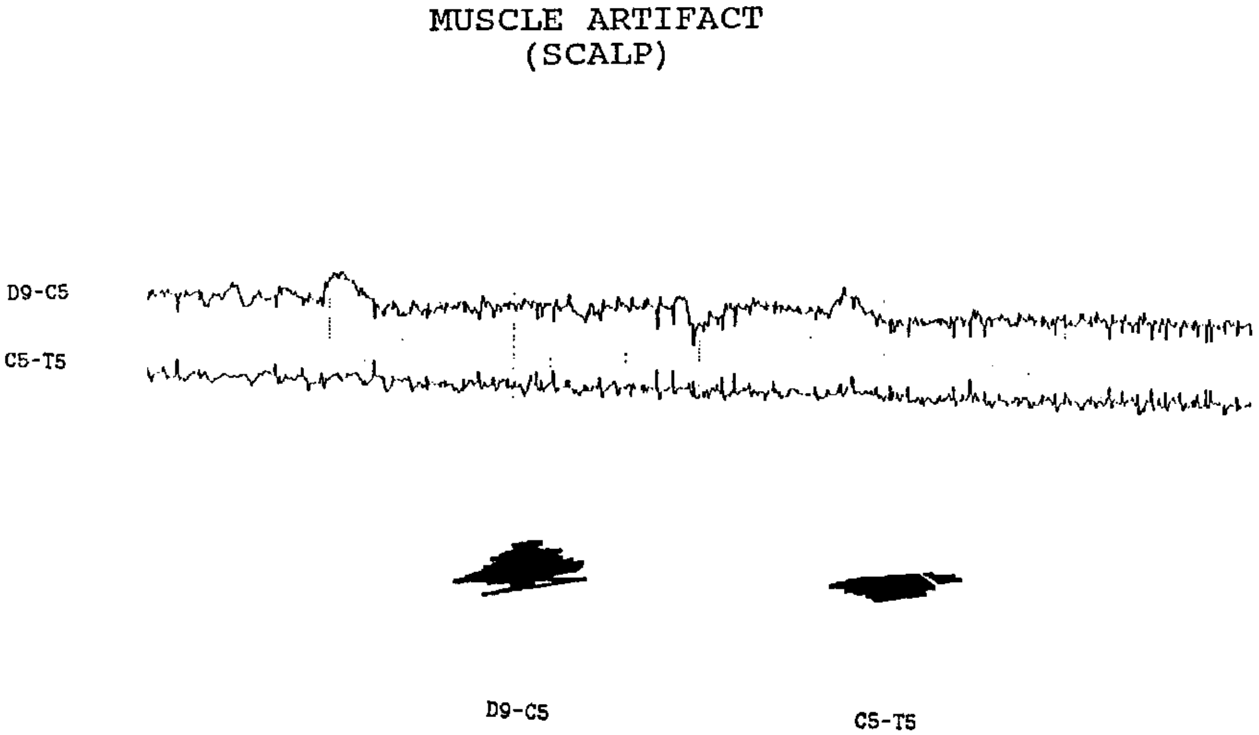

FIG. 3. EEG and phase-space trajectory of scalp EEG with muscle artifact.

epochs. Even with traces of this short duration, succeeding trajectories can vary dramatically, emphasizing the ability of the brain-EEG system to change its state in a very rapid manner.

Other investigators have reported systematic changes in trajectories associated with levels of anesthesia (Watt and Hameroff 1988). We felt that such changes might also be found in the EEG leading to seizure onset. Because of its

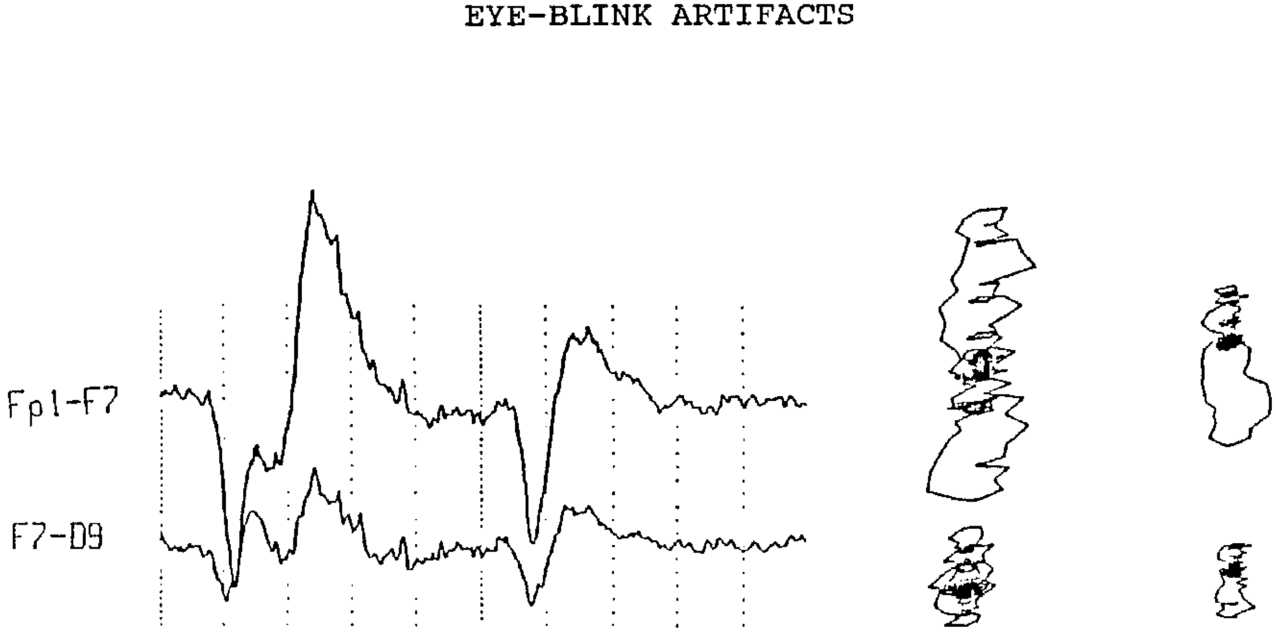

FIG. 4. EEG and phase-space trajectory of scalp EEG with eye-blink artifact.

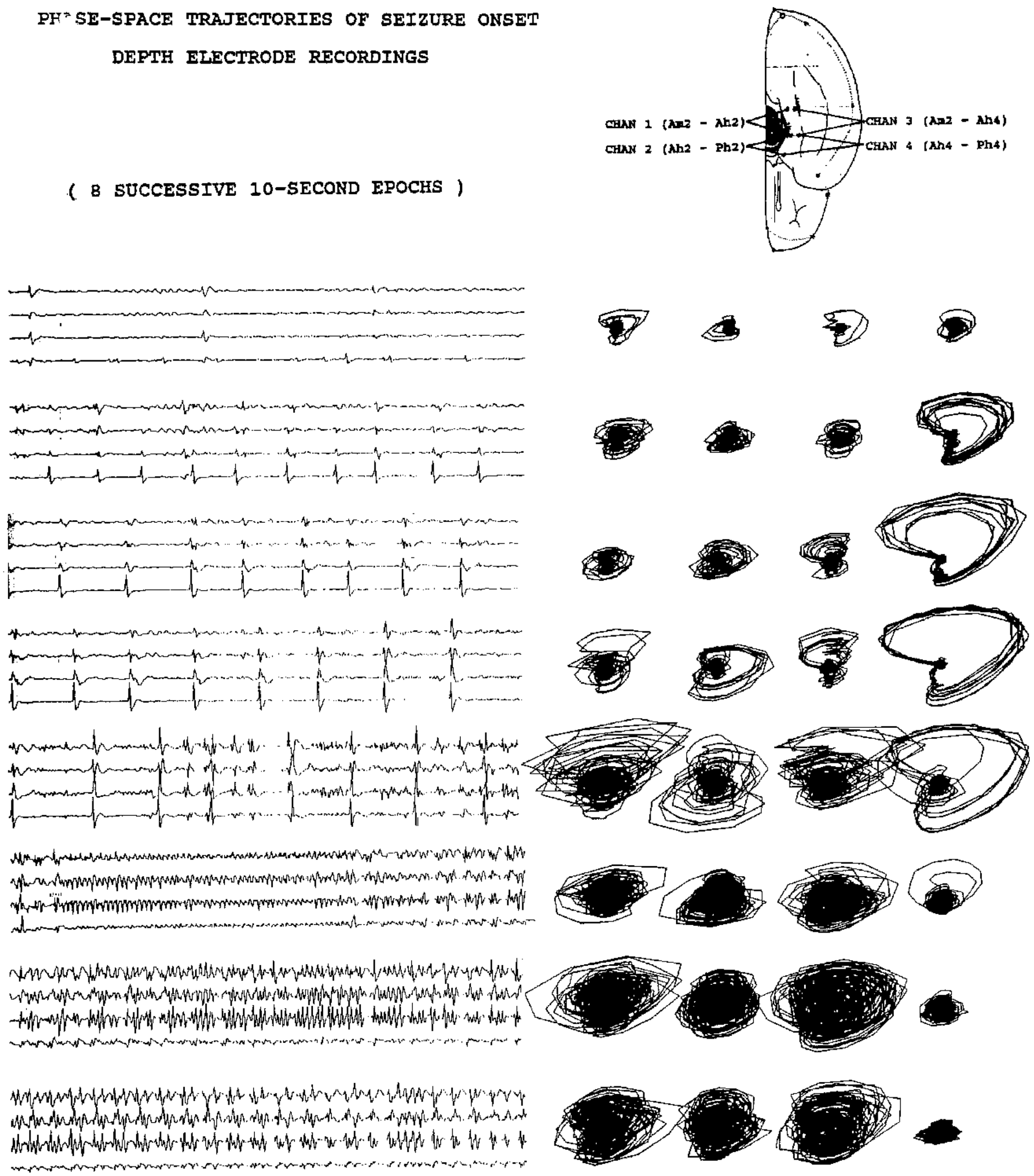

FIG. 5. Series of EEG and phase-space trajectories of a seizure onset recorded with limbic depth electrodes (8 successive 10-second epochs).

sensitivity to the state of a system, the phase-plane trajectory is potentially valuable for visualizing changes such as the transition from the preictal state to seizure onset in an epileptic. We decided to explore phase-plane trajectories associated with this transition, and to systematically describe the changes that were evident, both in the time-domain and in the phase-plane.

Considerable attention has recently been paid to alternative representations

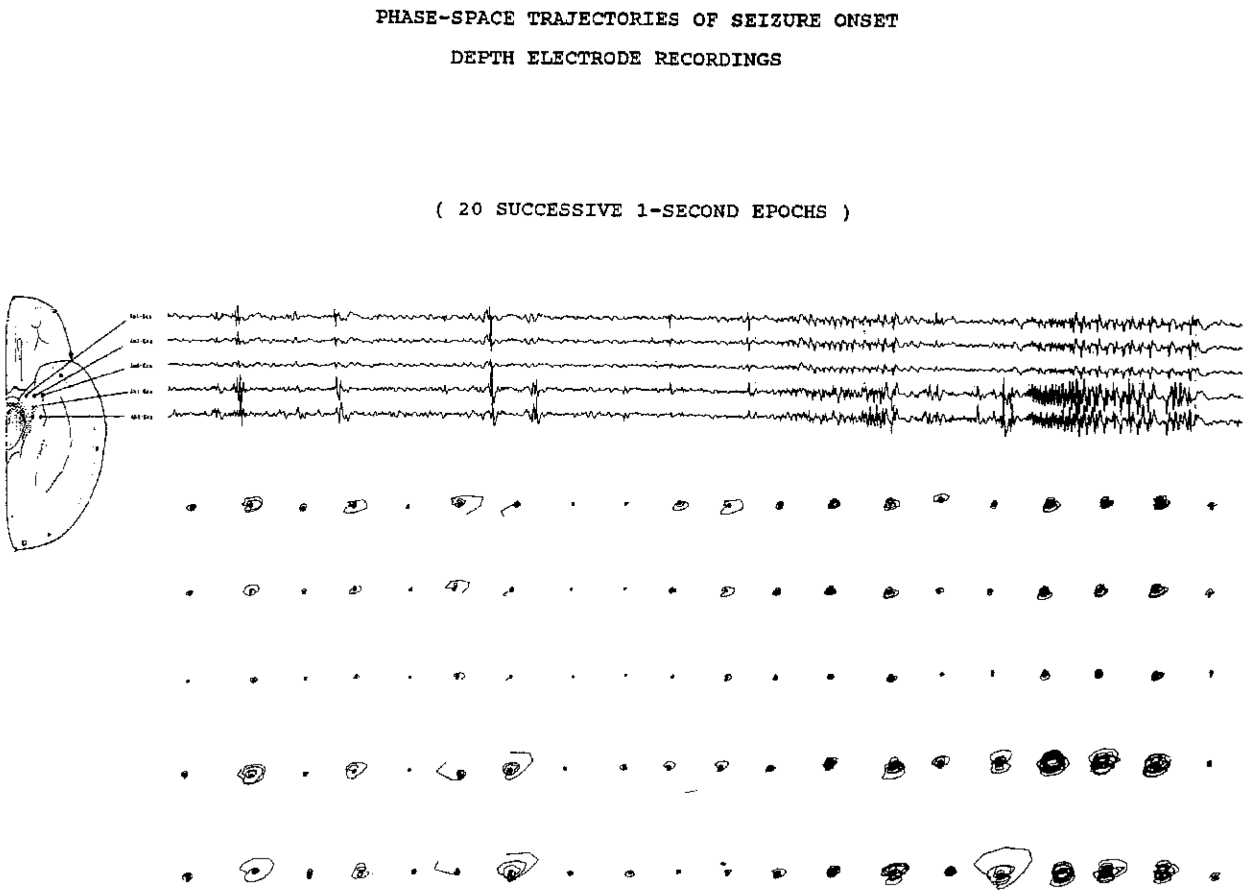

FIG. 6. Series of EEG and phase-space trajectories of a seizure onset recorded with limbic depth electrodes (20 successive 1-second epochs).

of EEG patterns, particularly those related to nonlinear dynamic analysis. It has been reported that dynamic changes in the EEG can precede overt seizure patterns, and are associated with the development of the seizure. (Velis 1990, Pijn 1990). Current work emphasizes mathematical computations that extract the "dimension" of the wave, and attempt to correlate changes in the dimension with electrical state transitions. The study of dimension is a discipline in itself, and a considerable literature is emerging relating to the computation and interpretation of EEG dimension values (Mayer-Kress and Layne, 1987). We note, however, that historically, mathematical manipulations of the EEG have failed to give way to visual inspection as a primary clinical tool. We were therefore interested in preserving the aspect of visual inspection, and applying it to the alternative representation provided by phase-plane.

We were interested in the possible value of this visual tool, which is based on current concepts from nonlinear dynamic analysis, but has the advantage of providing detailed information that can be scrutinized by the human eye. Since these trajectories may provide a new way to characterize, describe, and quantify EEG dynamics, we wished to evaluate their possible usefulness in clinical epilepsy.

This paper presents the results of visual evaluation of EEG patterns preceding seizures, with regard to important signal characteristics. We gave an experienced epileptologist (HHM) an opportunity to scrutinize phase-plane trajectories systematically gathered from EEGs showing epileptic seizures, and to develop visual criteria that might be used to distinguish the traces. Special attention was paid to the transition from pre-ictal to ictal EEG.

METHOD OF PHASE-PLANE REPRESENTATION

The exact choice of parameters for a phase-plane trajectory is a matter of some debate, and a variety of methods are available. While all are mathematically interchangeable, the choice should be based on practical considerations related to the signals at hand, and the requirements of a particular study. Most investigators base the trajectory on one axis that is the signal amplitude, and another that relates to the change in the signal. In this project, phase-plane trajectories were generated by plotting EEG amplitude as the vertical axis, and the first difference (derivative) as the horizontal axis. Other investigators have used a fixed delay to define the "change" axis, and this delay ranges from about 10 to 100 milliseconds. Some researchers choose a different delay for each epoch, basing the delay value on considerations of the autocorrelation or frequency spectrum of the signal. Other investigators not only choose a variable delay, but also use a projection of the trajectory that is based on multidimensional, mathematical considerations (Principe et al. 1990).

We have evaluated such displays, and believe that display consistency and reliable sample-to-sample comparisons should outweigh other theoretical considerations. It is impossible to find a single delay value that adequately reveals the range of frequencies and transients encountered in EEG, particularly in epilepsy. In addition, if the delay is different for different epochs, it is difficult to compare plots, since any similarities that do exist will be obscured by the changes in the display parameters. Similarly, changes in display parameters may introduce misleading differences in plots, even for components that do not change. Due to these difficulties in using variable length delays, we have chosen the first derivative for all plots. This method has the advantage of providing a single, simple representation for all epochs, and of being relatively easy to compute and to understand.

Phase-plane trajectories can be shown to adequately capture the dynamics of a system when they have enough "dimensionality" to represent the underlying system. Mathematically, a two-dimensional plot conveys all of the information in a system if it has a "chaotic" dimension of two or less. Intuitively, a low-dimensional system is more ordered and has less random behavior than a high-dimensional system. However, this notion has rigorous meaning only for very long segments of data, and the EEG typically does not remain stationary long enough for an analytically correct dimension calculation. In the short-term, we believe that epileptiform activity such as sharp-waves or rhythmic activity can be thought of as derived from low-dimensional processes, even though the dimension cannot be exactly calculated. When the EEG does manifest sustained epileptiform activity, as in the case of a seizure, calculations have shown that the dimension of the signal is relatively low, approaching or even going below a value of 2 (Babloyantz and Destexhe 1986). This contrasts with the normal EEG which, depending on state of consciousness and task involvement, generally ranges from 3 to 5 (Babloyantz et al., 1985, Rapp et. al. 1989).

INITIAL OBSERVATIONS

Phase-plane trajectories provide a compact graphical representation of EEG epochs, and are easy to compute and display. They emphasize certain attributes of the EEG signals, and de-emphasize or even obscure others. High-amplitude, fast activity is clearly distinguished by the presence of large, conspicuous trajectories. Phase relationships between channels can be easily seen, as long as the features emerge clearly from the central portion of the trajectory. Specific details of signal morphology are effectively enhanced, and converted into two-dimensional shapes that are easily recognized. Sharp transients, EMG activity, and eye-related artifacts are easily seen and distinguished. It is often easy to distinguish the baseline as a small center of activity at the origin of the plot, and subtle changes in baseline amplitude are easily seen as changes in the size of this center.

Although phase-plane trajectories emphasize certain important characteristics of signal morphology, they obscure other features known to be valuable in clinical EEG. The absolute time reference is lost, so that trends within an epoch will not be clearly visible. If, for example, rhythmic activity is steadily increasing throughout a 10-second record, a phase-plane trajectory of that record will show the rhythmic activity, but will not clearly reveal the time-course of the changes in the activity. Also, smaller waves can be lost in noise or larger-amplitude activity. In particular, a short burst of noise or other interfering activity can obscure a signal in the phase-plane trajectory, even if the two are clearly separated in a time-series plot.

Overall, low-voltage rhythmic activity is generally difficult or impossible to see, even in a relatively clean sample. Finally, critical time relationships, such as a gradual increase in the repetition rate of sharp-waves, cannot be discerned in a trajectory, since all the waves are overlaid on the plot. Interchannel timing can be similarly obscured, making it impossible to judge synchrony or cause/effect relationships.

PROCEDURE

We attempted a systematic evaluation of phase-plane trajectories made from EEGs associated with epileptic seizures. This study was broken into three basic parts: initial evaluation trial, two classification trials, and a final classification trial. In each part, 15 seizures were used. Each seizure was represented by two 10-second samples. One sample was taken from an epoch 2 minutes before the EEG onset of the seizure ("interictal"). Another sample was taken from an epoch that preceded the epoch containing the EEG onset ("preictal"). Thus, although this epoch did not contain the overt signs of seizure initiation, it might contain visual cues of impending seizure activity such as subtle changes in baseline, or barely perceptible rhythms. Thus, the "gold standard" for this study was the EEG seizure onset as defined in the routine clinical practice of our epilepsy section.

A total of 60 seizures were thus inspected, from a variety of scalp and invasive electrode recordings. All seizures were selected at random from awake recordings taken from the 24-hour epilepsy monitoring unit at the Cleveland Clinic. No individual patient contributed more than 2 seizures to the study. Invasive recordings included bilateral limbic depth, subdural grids, and foramen ovale placements. None of the epochs included major artifacts or noise, although typical seizure-related EMG signals were allowed in the scalp-recorded EEGs. The 60 seizures were studied in 4 groups of 15, as described below.

In order to compare phase-plane trajectories with conventional time-series plots, both types of printouts were prepared for each epoch. The learning samples were provided with full annotations and labels. For the classification trials, records were identified on the prints with a code number that was assigned to the patient identification and seizure number using a list prepared by an uninvolved third party in a double-blind fashion. Thus, although the electroencephalographer (EEGer) knew that a pair of epochs went together and knew the electrode identification, he did not know the identity of the patient, or which of the epochs was interictal or preictal.

RESULTS

In the learning trial (L1), the EEGer was allowed to inspect 15 fully annotated records in both time-domain and phase-domain, and developed a set of classification criteria that emerged from a systematic evaluation. These criteria are shown in Table 1. In a given set of records, these criteria, when present, were found to distinguish the interictal records from the preictal records. It is evident that the time-series plots provided a richer set of criteria than the phase-plane trajectories. The primary reason for this is that the time-series provided the

TABLE 1. *Judgement Criteria for Time and Phase Plots.*

Time-Series EEG
  Evolving patterns:
    "arousal"—alpha extinguished
    "drowsy"—slowing, reduced fast activity
    "epileptiform"—abnormal fast background rhythm
  Visible transition in overall rhythm
  Decremental activity
  Rhythmic slowing
  Fast activity
  Small or large sharpwaves
    single or repetitive
  More vs. less "normal" (background)

Phase-Plane Trajectories
  Decremental activity ("small center")
  Small or large sharpwaves ("large loops")
    single vs. repetitive ("one vs. many") ("concentric rings")
  Repetitive rhythm ("cohesive ball")

additional evidence of transitional or changing patterns, and these were obscured in the phase domain.

In each of the two classification trials (C1 and C2), the EEGer attempted to classify 15 records provided without annotations, but identified only by the blind index numbers. The records were classified once using the time-domain only, and again several days later using the phase-domain only. Results of the classifications are shown in Table 2. This table shows the number of correctly classified seizures using time-domain only, phase-domain only, as well as the number of records correctly classified on both methods, and the number correctly classified on either method. With each value, we provide a p value based on a binary choice model.

The binary model is based on the model of a coin toss in which p(correct)

TABLE 2. *Results of Classification Trials.*

Trial	EEG	Phase	Both	Either
C1	12 (0.01)	6 (0.30)	3 (0.02)	14 (0.02)
C2	11 (0.02)	7 (0.50)	4 (0.06)	14 (0.02)

Legend: EEG: number of epochs correctly classified based on EEG; Phase: number correctly classified based on Phase-plane; Both: number of epochs correctly classified on both EEG and Phase; Either: number of epochs correct on either EEG or phase; p-values from a binary choice model with p(correct) = 0.5 for EEG and Phase; p(correct) = 0.25 for Both; p(correct) = 0.75 for Either.

TABLE 3. *Results of Classification Trial C1.*

Sz	Electrodes	EEG	Criteria	Phase	Criteria
1	scalp	yes	rhythmic slow	no	none
2	scalp	no	sm. sh. w. rep. rhythm	yes	decrem.
3	scalp	yes	vis. onset	yes	recur. sh. w.
4	f. ovale	no	lg. sh. w. rhyth. slow	no	rep. rhythm
5	subdural	yes	rep. rhythm decrem.	no	sm. sh. w. decrem.
6	subdural	yes	fast rhythm	no	rep. rhythm
7	scalp	no	alpha change	yes	decrem.
8	subdural	yes	decrem.	no	lg. sh. w.
9	f. ovale	yes	fast rhythm	no	rep. sh. w.
10	subdural	yes	vis. onset	yes	rep. rhythm
11	depth	yes	transition	yes	rep. rhythm lg. sh. w.
12	scalp	no	rhyth. slow	yes	decrem.
13	scalp	yes	fast rhythm	no	decrem.
14	scalp	yes	transition decrem.	no	decrem.
15	subdural	yes	rep. rhythm sm. sh. w.	no	decrem. sm. sh. w.

Legend: yes: correctly classified; no: not correctly classified; sm. sh. w.: small sharp wave; lg. sh. w.: large sharp wave; rep. rhythm: repetitive rhythm; vis. onset: visible onset; decrem.: decremental activity.

is 0.5 for the EEG and Phase judgements, which is equivalent to tossing one head in one toss. For the Both case, p(correct) is 0.25, which is equivalent to tossing 2 heads in two tosses. For the Either case, p(correct) is 0.75, which is the probability of tossing either a head and a tail, or a tail and head, or two heads, in two tosses.

It is seen that the EEG records provided classification at better than a chance level, while the phase-plane records did not. In addition, the degree of agreement between the two methods was low, as shown by the small number of trials on which the seizures were correctly classified on both. In contrast, since the two methods were generally in disagreement, the number of seizures correctly classified on one or the other was high. For the first classification trial C1, the specific results for each seizure and criteria are shown for illustration in Table 3.

After the classification trial C1, the records were reviewed along with the correct identification, in an attempt to provide direct feedback to the EEGer, and to revise the judgement criteria. The second classification trial C2 was conducted with this additional information. Specifically, more attention was paid to baseline amplitude as revealed by the size of the inner spot on the phase plots. This feature was used to indicate possible electrodecremental activity in the epoch prior to seizure onset. The results of C2 were essentially the same as C1, indicating that the initial experience was repeatable, and that no significant change was introduced by the additional information.

The results of C2 were reviewed before the final classification trial (C3), so that this final trial would have the benefit of the previous experience. In C3, the

EEGer used both the time-domain and phase-plane plots together, and attempted to use the combined information to classify the EEGs. Theoretically, since the two methods were generally in disagreement, if it were possible to mediate between them in an optimal fashion, correct judgements could be made significantly better than chance. It was not possible to develop a systematic way to combine the information, however, since there was no rationale for deciding which, if either, of the indicators could be deemed more trustworthy. In this trial, the number of seizures correctly classified was 9, corresponding to a p value of 0.2, which is not significant.

Therefore, the additional phase-plane information in this exercise was found to confuse judgements that had been made more accurately using EEG alone. While this is not a conclusive finding, it would appear that the phase-plane trajectories do not provide an obviously useful adjunct to traditional EEGs when used in this context. This does not preclude the possibility of developing a useful approach to combining these methods. However, it does indicate that such a method was not intuitively obvious in this first attempt.

CONCLUSION

It was found that phase-plane trajectories emphasize different signal characteristics than time-series plots, possibly leading to different conclusions. The phase plane can precisely show background activity, signal morphology, and subtle variations in size and slope of fast activity. However, phase plots can lose information about underlying rhythm, short-term changes, or evolving activity. In addition, sudden changes or transition patterns that are clearly evident in raw waveforms can be lost in the phase-plane trajectory. Individual differences may overshadow the immediate benefits inherent in the new information.

In an initial attempt to exploit this new information, phase-plane plots seemed to confuse rather than clarify decisions. Despite the negative result of this initial study, our experience sheds light on possible future applications of phase-plane trajectories in the study of dynamic brain processes.

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