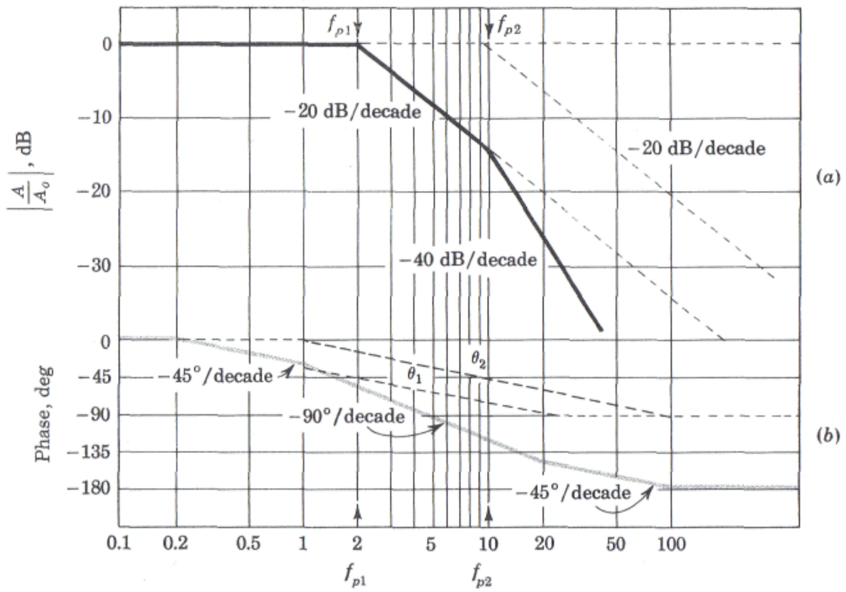

Fig. 12-6 (a) The idealized Bode amplitude plots for a two-pole transfer function. The individual asymptotes for each pole are shown dashed, and the resultant is drawn as a solid continuous broken-line plot. (b) Phase-response Bode plot for (a).

The Dominant Pole If in Eq. (12-16) f_{p1} is much smaller than f_{p2}, the above discussion in connection with Fig. 12-6 indicates that the upper 3-dB frequency is given approximately by f_{p1}. If $f_{p2} = 4f_{p1}$, an exact plot indicates (Prob. 12-7) that the 3-dB frequency is only 6 percent smaller than f_{p1}. We conclude that *if a transfer function has several poles determining the high-frequency response, if the smallest of these is f_{p1} and if each other pole is at least two octaves higher, then the amplifier behaves essentially as a single-time-constant circuit whose 3-dB frequency is f_{p1}.* The frequency f_{p1} is called the *dominant pole.*

12-5 STEP RESPONSE OF AN AMPLIFIER

An alternative criterion of amplifier fidelity is the response of the amplifier to a particular input waveform. Of all possible available waveforms, the most generally useful is the step voltage. In terms of a circuit's response to a step, the response to an arbitrary waveform may be written in the form of the superposition integral. Another feature which recommends the step voltage is the fact that this waveform is one which permits small distortions to stand out clearly. Additionally, from an experimental viewpoint, we note that

excellent pulse (a short step) and square-wave (a repeated step) generators are available commercially.

As long as an amplifier can be represented by a dominant pole, the correlation between its frequency response and the output waveshape for a step input is that given below. Quite generally, even for more complicated amplifier circuits, there continues to be an intimate relationship between the distortion of the leading edge of a step and the high-frequency response. Similarly, there is a close relationship between the low-frequency response and the distortion of the flat portion of the step. We should, of course, expect such a relationship, since the high-frequency response measures essentially the ability of the amplifier to respond faithfully to rapid variations in signal, whereas the low-frequency response measures the fidelity of the amplifier for slowly varying signals. An important feature of a step is that it is a combination of the most abrupt voltage change possible and of the slowest possible voltage variation.

Rise Time The response of the low-pass circuit of Fig. 12-2 to a step input of amplitude V is exponential with a time constant R_2C_2. Since the capacitor voltage cannot change instantaneously, the output starts from zero and rises toward the steady-state value V, as shown in Fig. 12-7. The output is given by

$$v_o = V(1 - \epsilon^{-t/R_2C_2}) \tag{12-19}$$

The time required for v_o to reach one-tenth of its final value is readily found to be $0.1R_2C_2$, and the time to reach nine-tenths its final value is $2.3R_2C_2$. The difference between these two values is called the *rise time* t_r of the circuit and is shown in Fig. 12-7. The time t_r is an indication of how fast the amplifier can respond to a discontinuity in the input voltage. We have, using Eq. (12-7),

$$t_r = 2.2R_2C_2 = \frac{2.2}{2\pi f_H} = \frac{0.35}{f_H} \tag{12-20}$$

Note that the rise time is inversely proportional to the upper 3-dB frequency. For an amplifier with 1 MHz bandpass, $t_r = 0.35\ \mu$s.

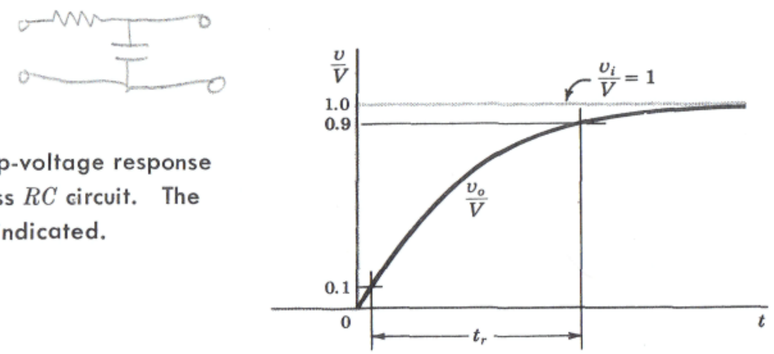

Fig. 12-7 Step-voltage response of the low-pass RC circuit. The rise time t_r is indicated.

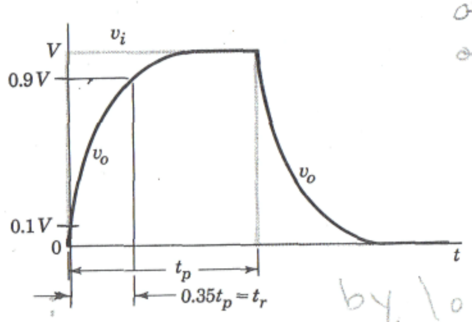

Fig. 12-8 Pulse response for the case $f_H = 1/t_p$.

high f distortion
by low pass filter
(pole)

Consider a pulse of width t_p. What must be the high 3-dB frequency f_H of an amplifier used to amplify this signal without excessive distortion? A reasonable answer to this question is: *Choose f_H equal to the reciprocal of the pulse width, $f_H = 1/t_p$.* From Eq. (12-20) we then have $t_r = 0.35t_p$. Using this relationship, the (shaded) pulse in Fig. 12-8 becomes distorted into the (solid) waveform, which is clearly recognized as a pulse.

Tilt or Sag If a step of amplitude V is impressed on the high-pass circuit of Fig. 12-1, the output is

$$v_o = V\epsilon^{-t/R_1C_1} \tag{12-21}$$

For times t which are small compared with the time constant R_1C_1, the response is given by

$$v_o \approx V\left(1 - \frac{t}{R_1C_1}\right) \tag{12-22}$$

From Fig. 12-9 we see that the output is tilted, and the percent tilt, or sag, in time t_1 is given by

$$P \equiv \frac{V - V'}{V} \times 100 = \frac{t_1}{R_1C_1} \times 100\% \tag{12-23}$$

It is found[6] that this same expression is valid for the tilt of each half cycle of a symmetrical square wave of peak-to-peak value V and period T provided

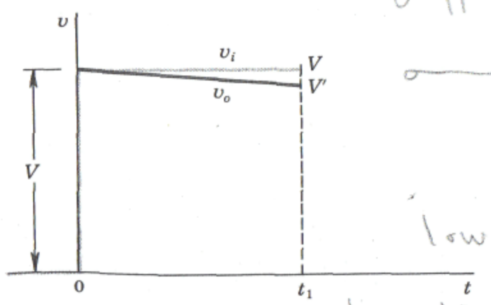

Fig. 12-9 The response v_o, when a step v_i is applied to a high-pass RC circuit, exhibits a tilt.

low f distortion
by high pass filter.

that we set $t_1 = T/2$. If $f = 1/T$ is the frequency of the square wave, then, using Eq. (12-3), we may express P in the form

$$P = \frac{T}{2R_1C_1} \times 100 = \frac{1}{2fR_1C_1} \times 100 = \frac{\pi f_L}{f} \times 100\% \tag{12-24}$$

Note that the tilt is directly proportional to the lower 3-dB frequency. If we wish to pass a 50-Hz square wave with less than 10 percent sag, then f_L must not exceed 1.6 Hz.

Square-wave Testing An important experimental procedure (called *square-wave testing*) is to observe with an oscilloscope the output of an amplifier excited by a square-wave generator. It is possible to improve the response of an amplifier by adding to it certain circuit elements,[1] which then must be adjusted with precision. It is a great convenience to be able to adjust these elements and to see simultaneously the effect of such an adjustment on the amplifier output waveform. The alternative is to take data, after each successive adjustment, from which to plot the amplitude and phase responses. Aside from the extra time consumed in this latter procedure, we have the problem that it is usually not obvious which of the attainable amplitude and phase responses corresponds to optimum fidelity. On the other hand, the step response gives immediately useful information.

It is possible, by judicious selection of two square-wave frequencies, to examine individually the high-frequency and low-frequency distortion. For example, consider an amplifier which has a high-frequency time constant of 1 μs and a low-frequency time constant of 0.1 s. A square wave of half period equal to several microseconds, on an appropriately fast oscilloscope sweep, will display the rounding of the leading edge of the waveform and will not display the tilt. At the other extreme, a square wave of half period approximately 0.01 s on an appropriately slow sweep will display the tilt, and not the distortion of the leading edge. Such a waveform is indicated in Fig. 12-10.

It should *not* be inferred from the above comparison between steady-state and transient response that the phase and amplitude responses are of no importance at all in the study of amplifiers. The frequency characteristics are useful for the following reasons. In the first place, much more is known generally about the analysis and synthesis of circuits in the frequency domain than in the time domain, and for this reason the design of coupling networks is often done on a frequency-response basis. Second, it is often possible to arrive at least at a qualitative understanding of the properties of a circuit from a study of the steady-state-response in circumstances where transient calculations are extremely cumbersome. Third, compensating an amplifier against unwanted oscillations (Chap. 14) is accomplished in the frequency domain. Finally, it happens occasionally that an amplifier is required whose characteristics are specified on a frequency basis, the principal emphasis being to amplify a sine wave.

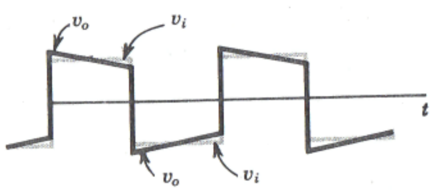

Fig. 12-10 A square-wave (shaded) input signal is distorted by an amplifier with a lower 3-dB frequency f_L. The output (solid) waveform shows a tilt where the input is horizontal.

12-6 BANDPASS OF CASCADED STAGES

The high 3-dB frequency for n cascaded stages is f_H^* and equals the frequency for which the overall voltage gain falls 3 dB to $1/\sqrt{2}$ of its midband value. To obtain the overall transfer function of *noninteracting* stages, the transfer gains of the individual stages are multiplied together. Hence, if each stage has a dominant pole and if the high 3-dB frequency of the ith stage is f_{Hi}, where $i = 1, 2, \ldots, n$, then f_H^* can be calculated from the product

$$\frac{1}{\sqrt{1+(f_H^*/f_{H1})^2}} \cdots \frac{1}{\sqrt{1+(f_H^*/f_{Hi})^2}} \cdots \frac{1}{\sqrt{1+(f_H^*/f_{Hn})^2}} = \frac{1}{\sqrt{2}} \qquad (12\text{-}25)$$

For n stages with identical upper 3-dB frequencies we have

$$f_{H1} = f_{H2} = \cdots = f_{Hi} = \cdots = f_{Hn} = f_H$$

Thus f_H^* is calculated from

$$\left[\frac{1}{\sqrt{1+(f_H^*/f_H)^2}}\right]^n = \frac{1}{\sqrt{2}}$$

to be

$$\frac{f_H^*}{f_H} = \sqrt{2^{1/n}-1} \qquad (12\text{-}26)$$

For example, for $n = 2$, $f_H^*/f_H = 0.64$. Hence two cascaded stages, each with a bandwidth $f_H = 10$ kHz, have an overall bandwidth of 6.4 kHz. Similarly, three cascaded 10-kHz stages give a resultant upper 3-dB frequency of 5.1 kHz, etc.

If the low 3-dB frequency for n identical noninteracting cascaded stages is f_L^*, then, corresponding to Eq. (12-26), we find

$$\frac{f_L^*}{f_L} = \frac{1}{\sqrt{2^{1/n}-1}} \qquad (12\text{-}27)$$

We see that a cascade of stages has a lower f_H and a higher f_L than a single stage, resulting in a shrinkage in bandwidth.

If the amplitude response for a single stage is plotted on log-log paper, the resulting graph will approach a straight line whose slope is 6 dB per octave both at the low and at the high frequencies, as indicated in Fig. 12-3. For an

n-stage amplifier it follows that the amplitude response falls $6n$ dB per octave, or equivalently, $20n$ dB per decade.

Interacting Stages If in a cascade of stages the input impedance of one stage is low enough to act as an appreciable shunt on the output impedance of the preceding stage, then it is no longer possible to isolate stages. Under these circumstances individual 3-dB frequencies for each stage cannot be defined. However, when the overall transfer function of the cascade is obtained (Sec. 12-10), it is found to contain n poles (in addition to k zeros). If the pole frequencies are $f_1, \ldots, f_2, \ldots, f_n$, then the high 3-dB frequency of the entire cascade f_H^* is given by Eq. (12-25) (with f_{Hi} replaced by f_i), provided that the zero frequencies are much higher than the pole frequencies (Prob. 12-14).

If the cascade has a dominant pole f_D which is much smaller than all other poles, all terms in the product in Eq. (12-25) may be neglected except the first. It then follows that $f_H = f_D$, or the high 3-dB frequency equals the dominant-pole frequency. (From here on we shall drop the asterisk on f_H^*.)

Consider now the situation discussed in Sec. 12-4, where there is a dominant frequency f_D, a second pole whose frequency is only two octaves away, and all other poles are at very much higher frequencies. Then Eq. (12-25) becomes

$$\frac{1}{\sqrt{1+(f_H/f_D)^2}}\,\frac{1}{\sqrt{1+(f_H/4f_D)^2}} = \frac{1}{\sqrt{2}} \qquad (12\text{-}28)$$

Since we expect the 3-dB frequency to be approximately equal to the dominant frequency, substitute $f_H = f_D$ into the second term in Eq. (12-28) to obtain

$$1 + \left(\frac{f_H}{f_D}\right)^2 = \frac{2}{1+(\frac{1}{4})^2} \qquad (12\text{-}29)$$

or

$$f_H = 0.94 f_D \qquad (12\text{-}30)$$

This calculation verifies that the high 3-dB frequency is less than 6 percent smaller than the dominant frequency provided that the next higher pole frequency is at least two octaves away.

If the pole frequencies are not widely separated, the result of Prob. 12-15 indicates that f_H is given (to within 10 percent) by

$$\frac{1}{f_H} = 1.1\sqrt{\frac{1}{f_1^2} + \frac{1}{f_2^2} + \cdots + \frac{1}{f_n^2}} \qquad (12\text{-}31)$$

If this equation is applied to the case considered above, $f_1 = f_D$ and $f_2 = 4f_D$ and all other poles much higher, the result is $f_H = 0.89 f_D$, in close agreement with Eq. (12-30). If Eq. (12-31) is applied to the case where $f_1 = f_2$ and all other poles are much higher, then $f_H = 0.65 f_1$ (instead of the exact value of $0.64 f_1$). For three equal poles, Eq. (12-31) yields $f_H = 0.53 f_1$ (instead of the exact value of $0.51 f_1$).

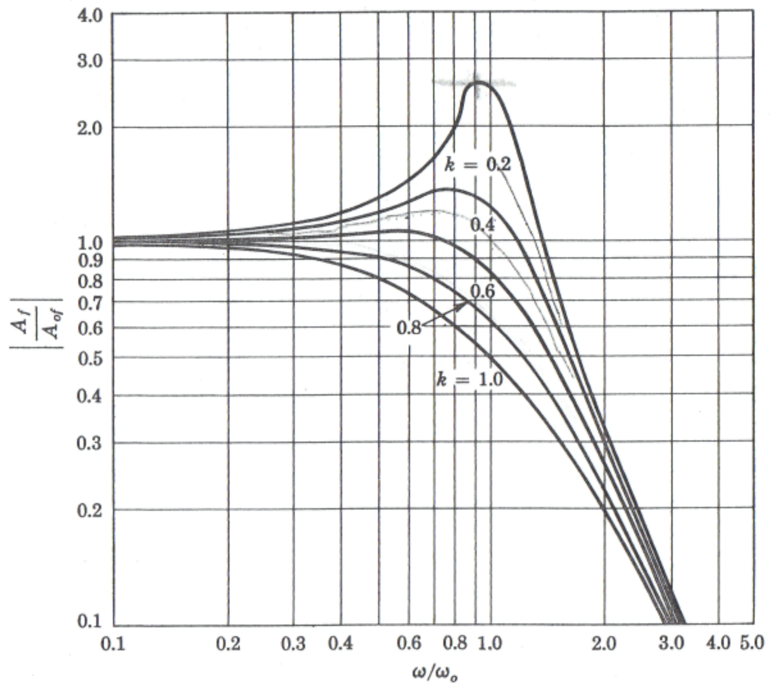

Fig. 14-5 Normalized plot of frequency response of a two-pole amplifier with feedback ($k = 1/2Q$).

For example, in Fig. 14-6 there is indicated one possible response to a voltage step. Note that the output overshoots its final value by 37 percent and oscillates before settling down to the steady-state voltage. For most applications such a violent response is not acceptable.

The important parameters of the waveform are indicated in Fig. 14-6 and are defined as follows:

Rise time = time for waveform to rise from 0.1 to 0.9 of its steady-state value

Delay time = time for waveform to rise from 0 to 0.5 of its steady-state value

Overshoot = peak excursion above the steady-state value

Damped period = time interval for one cycle of oscillation

Settling time = time for response to settle to within $\pm P$ percent of the steady-state value (P specified for a particular application, say $P = 0.1$)

Analytical expressions for the response of the amplifier to a step of amplitude V is obtained by setting $V_i(s) = V/s$ into Eq. (14-16) and solving for the inverse Laplace transform. Recalling from Eq. (14-17) that $Q = 1/2k$, the poles, given in Eq. (14-12), can be put into the form

$$s = -k\omega_o \pm \omega_o \sqrt{k^2 - 1} \tag{14-21}$$

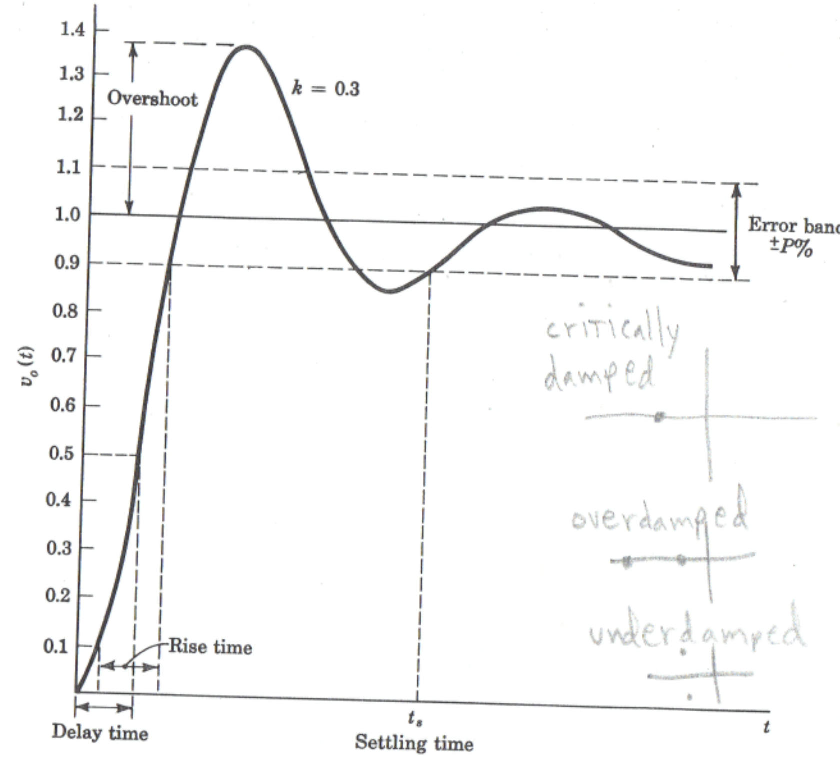

Fig. 14-6 The step response of a two-pole feedback amplifier for a damping factor $k = 0.3$.

If $k = 1$, the two poles coincide, corresponding to the *critically damped* case. If $k < 1$, the poles are complex conjugates, corresponding to an *underdamped* condition, where the response is a sinusoid whose amplitude decays with time. If $k > 1$, both poles are real and negative, corresponding to an *overdamped* circuit, where the response approaches its final value monotonically (without oscillation). For the underdamped case it is convenient to introduce the damped frequency

$$\omega_d \equiv \sqrt{1 - k^2}\,\omega_o \tag{14-22}$$

and the response $v_o(t)$ to a step of magnitude V into an amplifier of midband gain A_{of} is given by the following equations:

Critical damping, $k = 1$:

$$\frac{v_o(t)}{VA_{of}} = 1 - (1 + \omega_o t)\epsilon^{-\omega_o t} \tag{14-23}$$

Overdamped, $k > 1$:

$$\frac{v_o(t)}{VA_{of}} = 1 - \frac{1}{2\sqrt{k^2 - 1}}\left(\frac{1}{k_1}\epsilon^{-k_1\omega_o t} - \frac{1}{k_2}\epsilon^{-k_2\omega_o t}\right) \tag{14-24}$$

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